

Research Article

A Heterogeneous Multiattribute Group Decision-Making Method Based on Intuitionistic Triangular Fuzzy Information

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How to aggregate decision information in heterogeneous multiattribute group decision making (H MAGDM) is vital. The aim of this paper is to develop an approach to aggregating decision data into intuitionistic triangular fuzzy numbers (ITFNs) for heterogeneous MAGDM problems with real numbers (RNs), interval numbers (INs), triangular fuzzy numbers (TFNs), trapezoidal fuzzy numbers (TrFNs), and triangular intuitionistic fuzzy number (TIFNs). Using the relative closeness of technique for order preference by similarity to ideal solution (TOPSIS) and geometry entropy method, we first present a general approach to aggregating heterogeneous information into ITFNs, which takes the group consistency of experts into account. Based on the collective intuitionistic triangular fuzzy decision matrix and extended TOPSIS, a multiple objective mathematical program is constructed to determine the optimal attribute weights. Subsequently, a new method to solve H MAGDM problems is presented based on the aforementioned discussion. A trustworthy service selection example is provided to verify the practicality and effectiveness of the proposed method.

1. Introduction

Group decision making (GDM) has been known as a popular method for finding the best alternative from a set of alternatives through aggregating decision information given in a group of experts, in which the evaluation of alternatives may involve multiple attributes including objective and subjective information [1–3]. Due to the limited cognition and preference of decision maker, it is hard for different attributes to use the same information format to express the evaluation. For instance, in an online seller evaluation, the service attitude of the seller is suited to be described by triangular fuzzy numbers (TFNs) since the service of the seller is generally stable, but sometimes it is excellent and sometimes bad. It is convenient to describe the shipping speed of seller with interval numbers (INs) since it is not fixed but fluctuates in a certain range. These types of GDM problems with multiple conflicting attributes whose values are given by

decision makers (DMs) may be represented in the form of multiple formats, such as real numbers (RNs), INs, TFNs, trapezoidal fuzzy numbers (TrFNs), and linguistic values (LVs), called heterogeneous multiattribute group decision making (H MAGDM) problems [4].

In this recent research, H MAGDM methods have been successfully applied to various fields, such as supply chain coordination [5], business processes [6], and software quality evaluation [7–9]. The key to tackling such problems is how to fuse various types of attribute values [10]. So far, many useful and valuable methods have been developed to study the fusion process of heterogeneous information, which can be roughly classified into three main categories [4, 10]. (1) The indirect approaches [11–14], in which the heterogeneous decision information given by DMs is converted into uniformed information by transformation methods. Wang and Cai [13] developed a generic distance-based VIKOR which can use aggregation function to convert heterogeneous

information into a uniform nonfuzzy degree and applied it to deal with emergency supplier selection. Using transformation function, Zhang et al. [14] transformed the multigranular linguistic decision matrices (LDMs) into uniform LDMs. Then, a new optimization consensus model was constructed for 2-Rank multigranular linguistic MAGDM problems. (2) The optimization-based approaches [5, 15–17], in which the heterogeneous information is integrated by constructing different multiple objective optimization models. Dong et al. [15] proposed a new complex and dynamic HMAGDM method to deal with the differences between individual sets of attributes and heterogeneous information. Zhang et al. [16] developed a HMAGDM method with aspirations information by combining the prospect theory and a biobjective intuitionistic fuzzy programming model. Yu et al. [17] incorporated risk attitude and preference deviation of experts into the mathematical programming models to solve the HMAGDM problems with RNs, INs, IFs, LVs, and TFNs. (3) The direct approaches [10, 18–21]. In the direct approach, the collective decision information is obtained by aggregating standardized individual decision information. Then the heterogeneous information is transformed into some comparable preference information. Yue and Jia [10] introduced a projection measure to aggregate decision information including IFNs and IVIFs. Yue [18] proposed a direct projection-based group decision-making methodology with RNs and INs. To overcome the irrationality of the classical projection formulae in RN and IN vector settings, Yue [19] presented a normalized projection measure and applied it to solve HMAGDM problems with RNs and INs. In order to integrate heterogeneously interrelated attributes in the HMAGDM problem, Das et al. [20] develop an Atanassov's intuitionistic fuzzy extended Bonferroni mean based on a strict t-conorm. Li et al. [21] proposed a new HMAGDM method using weighted power average operator to integrate the heterogeneous decision data.

These achievements have provided the foundation of the HMAGDM problems. It is noticed that methods [1, 5–8, 15–21] are on the basis of the hypothesis that the ratings provided by DMs are completely affirmative, neglecting the judgment subjectivity; thus, the impreciseness and uncertainty of original decision information cannot be captured. Methods [10–14] turned the heterogeneous information into a unified form of linguistic terms, which are subjective and cannot measure quantitatively and intuitively the uncertainty of attribute values. Intuitionistic fuzzy (IF) sets (IFSs) [22, 23] and interval-valued intuitionistic fuzzy sets (IVIFSs) [24] can be viewed as an effective tool to describe the uncertainty and ambiguity, which has led to the wide applications of IFSs and IVIFSs [25–27]. To fill these research gaps, many practical studies have been proposed to aggregate decision data into IFSs [22–24], which can be divided into two categories: (1) the methods for aggregating RNs into intuitionistic fuzzy number (IFN) based on Golden Section idea [28, 29], Minimax Criterion [30], and statistical theory [31]; (2) the methods for aggregating RNs or INs into interval-valued intuitionistic fuzzy number (IVIFN) [1] based on Minimax Criterion [32], linear transformation [33], and mean and standard deviation [34]. However, these methods [28–34] cannot be suitable for

HMAGDM problems. More recently, Xu et al. [35] presented a general method to aggregate decision information into IFN and applied it to select cloud computing service providers wherein the assessments take the form of RNs, INs, TFNs, TrFNs, and LVs. Combined with the relative closeness in technique for order preference by similarity to ideal solution (TOPSIS) and statistical theory [36], Wan et al. [37] developed a new general method to aggregate the attribute value vector into IVIFNs and used it for HMAGDM problem with RNs, INs, TFNs, and TrFNs.

The aforementioned methods [35, 37] have made deep discussions to HMAGDM problems based on aggregating decision data into IF information, but these aggregation techniques [28–35, 37] still suffer from some deficiencies. (1) They cannot deal with more complicated attribute values represented by triangular intuitionistic fuzzy number (TIFNs) and trapezoidal intuitionistic fuzzy numbers (TrIFNs). (2) They ignore the influence of different experts in aggregation process which may lead to unreasonable results. (3) The membership degree and nonmembership degree of integrated value in [28–35, 37] cannot reflect the distribution characteristics of the data like normal distribution. In many real decision situations, the evaluations of decision maker are based on a number of historical feedbacks on the corresponding attribute. Studies showed that the distribution of historical feedbacks is generally close to a normal distribution when the number of feedbacks is larger. It may be effective to use TFNs to model the integrated value instead of crisp value and interval-value since TFNs contain more information and are more consistent with normal distribution characteristics. Thus, when an assessment vector is aggregated into an IFN and IVIFN, the loss of information is likely to occur. Intuitionistic triangular fuzzy numbers (ITFNs) introduced by Liu and Yuan [38], as an extension of IFSs, can express more information from different dimension decision information [39] than IFNs and IVIFNs since its prominent characteristic is that the corresponding membership degree and nonmembership degree are described by TFNs [40]. Thus, ITFNs can not only depict the fuzzy concept of “good” or “excellent,” but also outstand the satisfaction and dissatisfaction information with the maximum probability and also recoup the deficiency due to the loss of the center of gravity in IVIFNs [41–43]. For instance, in a trustworthy seller selection example, the service attitude may be expressed by an ITFN $((0.4, 0.6, 0.7), (0.1, 0.2, 0.3))$, which contains two aspects of implication in the historical ratings of a seller: one is that users' satisfactory degree is between 0.4 and 0.7; the most possible satisfactory degree is 0.6; the other is that users' dissatisfactory degree is between 0.1 and 0.3; the most possible dissatisfactory degree is 0.2. Some theories and GDM methods based on ITFNs have been developed. Wang [40] defined score function and accuracy function to compare the ITFNs and developed several ITFN geometric aggregation operators. Wei [41] proposed the ITFN weighted averaging operator and ITFN ordered weighted averaging operator and applied them to solve GDM problems. To consider the interaction among attributes, Gao et al. [42] presented some ITFN aggregation operators with interaction. Yu and Xu [43] investigated a series of intuitionistic multiplicative triangular

fuzzy aggregation operators. Although these studies [38–43] focused on different aspects of ITFNs, it can only aggregate ITFNs. Therefore, to push ahead with the application of the above aggregations, it is necessary to aggregate multiple types of decision information into ITFNs, which is very interesting yet relatively sophisticated to dispose of.

To do that, this paper aims to propose a novel HMAGDM method based on ITFNs. The primary contributions of this paper can be illuminated briefly as follows.

(1) We first present an aggregation technology to aggregate heterogeneous information into TIFNs. Compared with existing methods [28–35, 37], the proposed aggregate technology has the following advantages. For more details, refer to Section 5.2.

- (i) A new elicitation of the support, opposite, and uncertain information based on distance is introduced, which can accommodate more complicated attribute values including TIFNs and TrIFNs since it just needs to calculate the distance from decision data to the maximum and minimum grade.
- (ii) A new construction approach of ITFN is presented by group consistency which not only takes into account expert's weight but can overcome the shortcoming of the hypothesis of the normal distribution.
- (iii) It can not only effectively avoid the loss of original information, but also reflect the distribution characteristics of the original decision data.

(2) A new similarity measure of ITFNs is developed and applied to construct a multiple objective linear programming to determine attribute weights in ITFN environment with incomplete information. The determination method of attribute weights can effectively avoid the subjectivity brought by the given attribute weights in advance.

(3) Based on the aforesaid provision, a new method to deal with HMAGDM problems with RNs, INs, TFNs, TrFNs, and TIFNs is proposed. The comprehensive evaluation value of the alternative is an ITFN, which preserves more useful information.

The remainder of this paper is set out as follows. Section 2 briefly introduces related basic concepts. Section 3 presents an approach to aggregating heterogeneous decision data into ITFNs. Section 4 builds a multiple objective linear programming model to determine attribute weights and propose a HMAGDM method. Section 5 provides a numerical example to illustrate the feasibility and reasonableness of the proposed method. Section 6 makes our conclusions.

2. Preliminary

In this section, some basic concepts of ITFN and distance measures are briefly described below.

2.1. Intuitionistic Triangular Fuzzy Number

Definition 1 (see [38]). A triangular fuzzy number (TFN) A is a special fuzzy set on a real number set R ; its membership function is defined by

$$F_A(x) = \begin{cases} \frac{x - t_l}{t_m - t_l}, & \text{if } t_l \leq x \leq t_m, \\ \frac{t_h - x}{t_h - t_m}, & \text{if } t_m \leq x \leq t_h, \\ 0, & \text{if otherwise,} \end{cases} \quad (1)$$

where $0 \leq t_l \leq t_m \leq t_h \leq 1$, t_l and t_h present the lower limit and upper limit of A , respectively, and t_m is the mode, which can be denoted as a triplet (t_l, t_m, t_h) .

Definition 2 (see [38]). Let X be a fixed set; $\mu_{\tilde{A}}(x) = (t_A^l(x), t_A^m(x), t_A^h(x))$ and $\nu_{\tilde{A}}(x) = (f_A^l(x), f_A^m(x), f_A^h(x))$ are TFNs defined on the unit interval $[0, 1]$; then an intuitionistic triangular fuzzy set \tilde{A} over X is defined as $\tilde{A} = \{(x, < \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >) \mid x \in X\}$ where the parameters $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ indicate, respectively, the membership degree and nonmembership degree of the element x in \tilde{A} , with the conditions $0 \leq t_A^h(x) + f_A^h(x) \leq 1$.

For convenience, we call $\tilde{\alpha} = ((t_A^l, t_A^m, t_A^h), (f_A^l, f_A^m, f_A^h))$ an intuitionistic triangular fuzzy number (ITFN), where

$$\begin{aligned} t_A^l, t_A^m, t_A^h &\in [0, 1], \\ f_A^l, f_A^m, f_A^h &\in [0, 1], \\ t_A^h + f_A^h &\in [0, 1]. \end{aligned} \quad (2)$$

It is clear that the largest and smallest ITFN are $\alpha^+ = ((1, 1, 1), (0, 0, 0))$ and $\alpha^- = ((0, 0, 0), (1, 1, 1))$, respectively.

Definition 3 (see [38]). Let $\tilde{\alpha}_1 = ((t_1^l, t_1^m, t_1^h), (f_1^l, f_1^m, f_1^h))$ and $\tilde{\alpha}_2 = ((t_2^l, t_2^m, t_2^h), (f_2^l, f_2^m, f_2^h))$ be two ITFNs; then the containment is

$$\begin{aligned} \tilde{\alpha}_1 &\subseteq \tilde{\alpha}_2 \\ \text{iff } t_1^l &\leq t_2^l, t_1^m \leq t_2^m, t_1^h \leq t_2^h, f_1^l \geq f_2^l, f_1^m \geq f_2^m \text{ and } f_1^h \geq f_2^h. \end{aligned} \quad (3)$$

Some arithmetic operations between ITFNs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are shown as below [40]:

$$\begin{aligned} (1) \quad \tilde{\alpha}_1 + \tilde{\alpha}_2 &= ((t_1^l + t_2^l - t_1^l t_2^l, t_1^m + t_2^m - t_1^m t_2^m, t_1^h + t_2^h - t_1^h t_2^h), \\ &\quad (f_1^l f_2^l, f_1^m f_2^m, f_1^h f_2^h)), \\ (2) \quad \lambda \tilde{\alpha} &= ((1 - (1 - t_1^l)^\lambda, 1 - (1 - t_1^m)^\lambda, 1 - (1 - t_1^h)^\lambda), \\ &\quad ((f_1^l)^\lambda, (f_1^m)^\lambda, (f_1^h)^\lambda)) \quad \lambda > 0. \end{aligned}$$

Definition 4 (see [41]). For a set of ITFNs $\tilde{\alpha}_i$ ($i = 1, 2, \dots, n$) that have associated an importance weight vector $\mathbf{w} = (w_1, w_2, \dots, w_n)^T$ with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. We call

$$\begin{aligned} ITFWA(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = & \sum_{i=1}^n w_i \tilde{\alpha}_i = \left(\left(1 - \prod_{i=1}^n (1 - t_i^l)^{w_i}, 1 - \prod_{i=1}^n (1 - t_i^m)^{w_i}, 1 - \prod_{i=1}^n (1 - t_i^h)^{w_i} \right), \right. \\ & \left. \left(\prod_{i=1}^n (f_i^l)^{w_i}, \prod_{i=1}^n (f_i^m)^{w_i}, \prod_{i=1}^n (f_i^h)^{w_i} \right) \right) \end{aligned} \quad (4)$$

an intuitionistic triangular fuzzy weighted average operator (ITFWA).

Definition 5. Let $\tilde{\alpha}_1 = ((t_1^l, t_1^m, t_1^h), (f_1^l, f_1^m, f_1^h))$ and $\tilde{\alpha}_2 = ((t_2^l, t_2^m, t_2^h), (f_2^l, f_2^m, f_2^h))$ be two ITFNs. A similarity measure $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2)$ between the ITFNs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ is defined as follows:

$$\begin{aligned} \vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2) = & 1 - \left[\frac{1}{12} (|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| \right. \\ & + |f_1^l - f_2^l| + |f_1^m - f_2^m| + |f_1^h - f_2^h|) + \frac{1}{2} \\ & \cdot \max(|t_1^l - t_2^l|, |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, \\ & |f_1^m - f_2^m|, |f_1^h - f_2^h|) \left. \right] \end{aligned} \quad (5)$$

Theorem 6. The similarity measure $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2)$ satisfies the following properties:

- (i) $0 \leq \vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2) \leq 1$.
- (ii) $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2) = 1$ if and only if $\tilde{\alpha}_1 = \tilde{\alpha}_2$.
- (iii) $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2) = \vartheta(\tilde{\alpha}_2, \tilde{\alpha}_1)$.
- (iv) If $\tilde{\alpha}_3$ is a ITFN and $\tilde{\alpha}_1 \subseteq \tilde{\alpha}_2 \subseteq \tilde{\alpha}_3$, then $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq \vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2)$ and $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq \vartheta(\tilde{\alpha}_2, \tilde{\alpha}_3)$.

Proof. It is easy to see that the proposed similarity measure $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2)$ meets the third property of Theorem 6. We only need to prove (i), (ii), and (iv).

For (i). By (2), we have

$$\begin{aligned} 0 & \leq |t_1^l - t_2^l| \leq 1, \\ 0 & \leq |t_1^m - t_2^m| \leq 1, \\ 0 & \leq |t_1^h - t_2^h| \leq 1, \\ 0 & \leq |f_1^l - f_2^l| \leq 1, \end{aligned}$$

$$0 \leq |f_1^m - f_2^m| \leq 1,$$

$$0 \leq |f_1^h - f_2^h| \leq 1.$$

(6)

It is easy to see that

$$\begin{aligned} 0 & \leq \frac{1}{12} (|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| + |f_1^l - f_2^l| \\ & + |f_1^m - f_2^m| + |f_1^h - f_2^h|) \leq \frac{1}{2} \\ 0 & \leq \frac{1}{2} \max(|t_1^l - t_2^l|, |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, \\ & |f_1^m - f_2^m|, |f_1^h - f_2^h|) \leq \frac{1}{2} \end{aligned} \quad (7)$$

Thus we get

$$\begin{aligned} 0 & \leq 1 - \left[\frac{1}{12} (|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| + |f_1^l - f_2^l| \right. \\ & - |f_2^l| + |f_1^m - f_2^m| + |f_1^h - f_2^h|) + \frac{1}{2} \max(|t_1^l - t_2^l|, \\ & |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, |f_1^m - f_2^m|, |f_1^h - f_2^h|) \left. \right] \\ & \leq 1 \end{aligned} \quad (8)$$

And then the inequality $0 \leq \vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2) \leq 1$ is established.

For (ii). When $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2) = 1$, if and only if

$$\begin{aligned} \frac{1}{12} (|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| + |f_1^l - f_2^l| + |f_1^m - f_2^m| \\ - |f_2^m| + |f_1^h - f_2^h|) = 0 \\ \frac{1}{2} \max(|t_1^l - t_2^l|, |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, |f_1^m - f_2^m|, \\ |f_1^h - f_2^h|) = 0 \end{aligned} \quad (9)$$

Apparently, it is easy to derive

$$\begin{aligned} |t_1^l - t_2^l| & = 0, \\ |t_1^m - t_2^m| & = 0, \\ |t_1^h - t_2^h| & = 0, \\ |f_1^l - f_2^l| & = 0, \\ |f_1^m - f_2^m| & = 0, \\ |f_1^h - f_2^h| & = 0. \end{aligned} \quad (10)$$

Thus we get $t_1^l = t_2^l$, $t_1^m = t_2^m$, $t_1^h = t_2^h$, $f_1^l = f_2^l$, $f_1^m = f_2^m$, $f_1^h = f_2^h$. And then $\tilde{\alpha}_1 = \tilde{\alpha}_2$.

For (iv). Since

$$\begin{aligned}
 t_1^l &\leq t_2^l \leq t_3^l, \\
 t_1^m &\leq t_2^m \leq t_3^m, \\
 t_1^h &\leq t_2^h \leq t_3^h, \\
 f_1^l &\geq f_2^l \geq f_3^l, \\
 f_1^m &\geq f_2^m \geq f_3^m, \\
 f_1^h &\geq f_2^h \geq f_3^h,
 \end{aligned} \tag{11}$$

we get

$$\begin{aligned}
 |t_1^l - t_2^l| &\leq |t_1^l - t_3^l|, \\
 |t_1^m - t_2^m| &\leq |t_1^m - t_3^m|, \\
 |t_1^h - t_2^h| &\leq |t_1^h - t_3^h|, \\
 |f_1^l - f_2^l| &\leq |f_1^l - f_3^l|, \\
 |f_1^m - f_2^m| &\leq |f_1^m - f_3^m|, \\
 |f_1^h - f_2^h| &\leq |f_1^h - f_3^h|.
 \end{aligned} \tag{12}$$

Based on the above inequalities, it is easy to derive

$$\begin{aligned}
 &|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| + |f_1^l - f_2^l| + |f_1^m - f_2^m| \\
 &- |f_2^h - f_3^h| \leq |t_1^l - t_3^l| + |t_1^m - t_3^m| + |t_1^h - t_3^h| \\
 &+ |f_1^l - f_3^l| + |f_1^m - f_3^m| + |f_1^h - f_3^h|,
 \end{aligned}$$

$$\begin{aligned}
 &\max(|t_1^l - t_2^l|, |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, |f_1^m - f_2^m|, \\
 &|f_1^h - f_2^h|) \leq \max(|t_1^l - t_3^l|, |t_1^m - t_3^m|, |t_1^h - t_3^h|, \\
 &|f_1^l - f_3^l|, |f_1^m - f_3^m|, |f_1^h - f_3^h|).
 \end{aligned} \tag{13}$$

Thus, it holds that

$$\begin{aligned}
 1 - \left[\frac{1}{12} (|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| + |f_1^l - f_2^l| \right. \\
 \left. + |f_1^m - f_2^m| + |f_1^h - f_2^h|) + \frac{1}{2} \max(|t_1^l - t_2^l|, \right. \\
 \left. |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, |f_1^m - f_2^m|, |f_1^h - f_2^h|) \right] \\
 \geq 1 - \left[\frac{1}{12} (|t_1^l - t_2^l| + |t_1^m - t_2^m| + |t_1^h - t_2^h| + |f_1^l - f_2^l| \right. \\
 \left. - |f_2^h - f_3^h| + |f_1^m - f_2^m| + |f_1^h - f_2^h|) + \frac{1}{2} \max(|t_1^l - t_2^l|, \right. \\
 \left. |t_1^m - t_2^m|, |t_1^h - t_2^h|, |f_1^l - f_2^l|, |f_1^m - f_2^m|, |f_1^h - f_2^h|) \right].
 \end{aligned} \tag{14}$$

Thus, $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq \vartheta(\tilde{\alpha}_1, \tilde{\alpha}_2)$. In the same way, it is proved that $\vartheta(\tilde{\alpha}_1, \tilde{\alpha}_3) \leq \vartheta(\tilde{\alpha}_2, \tilde{\alpha}_3)$. \square

2.2. Distance Measures. Hamming distance is easily processed and commonly used in the process of heterogeneous information processing. For INs $a = [a^l, a^u]$ and $b = [b^l, b^u]$ (or TFNs $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$, TrFNs $a = (a_1, a_2, a_3, a_4)$ and $b = (b_1, b_2, b_3, b_4)$, and TIFNs $a = ((a_1, a_2, a_3), u_a, v_a)$ and $b = ((b_1, b_2, b_3), u_b, v_b)$), the distance measures can be defined as follows [35, 44]:

$d(a, b)$

$$= \begin{cases} \frac{1}{2} (|a^l - b^l| + |a^u - b^u|), & \text{if } a, b \text{ are INs} \\ \frac{1}{3} (|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|), & \text{if } a, b \text{ are TFNs} \\ \frac{1}{4} (|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3| + |a_4 - b_4|), & \text{if } a, b \text{ are TrFNs} \\ \frac{1}{6} (|u_a a_1 - u_b b_1| + |u_a a_2 - u_b b_2| + |u_a a_3 - u_b b_3|) + |v_a a_1 - v_b b_1| + |v_a a_2 - v_b b_2| + |v_a a_3 - v_b b_3| + |a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|, & \text{if } a, b \text{ are TIFNs.} \end{cases} \tag{15}$$

3. A New Method for Heterogeneous MAGDM Problems

In this section, the presentation of heterogeneous MAGDM problems is given first. Then, an approach to aggregating heterogeneous information into ITFNs is developed.

3.1. Heterogeneous MAGDM Problems. For the sake of convenience, some symbols are introduced to characterize the heterogeneous MAGDM problems as follows:

(1) The group of DMs D_i ($i \in M = \{1, 2, \dots, m\}$).

(2) The set of attributes A_j ($j \in N = \{1, 2, \dots, n\}$). Denote the attribute weight vector by $\mathbf{w} = (w_1, w_2, \dots, w_n)$, where w_j

represents the weight of A_j such that $w_j \in [0, 1]$ ($j \in N$) and $\sum_{j=1}^n w_j = 1$.

(3) The set of alternatives S_k ($k \in P = \{1, 2, \dots, p\}$).

Since there are multiple formats of rating values, the attribute set $A = \{A_1, A_2, \dots, A_n\}$ is divided into four subsets $\widehat{A}_1 = \{A_1, A_2, \dots, A_{j_1}\}$, $\widehat{A}_2 = \{A_{j_1+1}, A_{j_1+2}, \dots, A_{j_2}\}$, $\widehat{A}_3 = \{A_{j_2+1}, A_{j_2+2}, \dots, A_{j_3}\}$, $\widehat{A}_4 = \{A_{j_3+1}, A_{j_3+2}, \dots, A_{j_4}\}$, and $\widehat{A}_5 = \{A_{j_4+1}, A_{j_4+2}, \dots, A_{j_5}\}$, where $1 \leq j_1 \leq j_2 \leq j_3 \leq j_4 \leq j_5 \leq n$, $\widehat{A}_t \cap \widehat{A}_k = \emptyset$ ($t, k = 1, 2, 3, 4, 5; t \neq k$), and $\bigcup_{t=1}^5 \widehat{A}_t = A$, \emptyset is an empty set. The rating values in the subsets \widehat{A}_e ($e = 1, 2, 3, 4, 5$) are in the form of RNs, INs, TFNs, TrFNs, and TIFNs, respectively. Denote the subscript

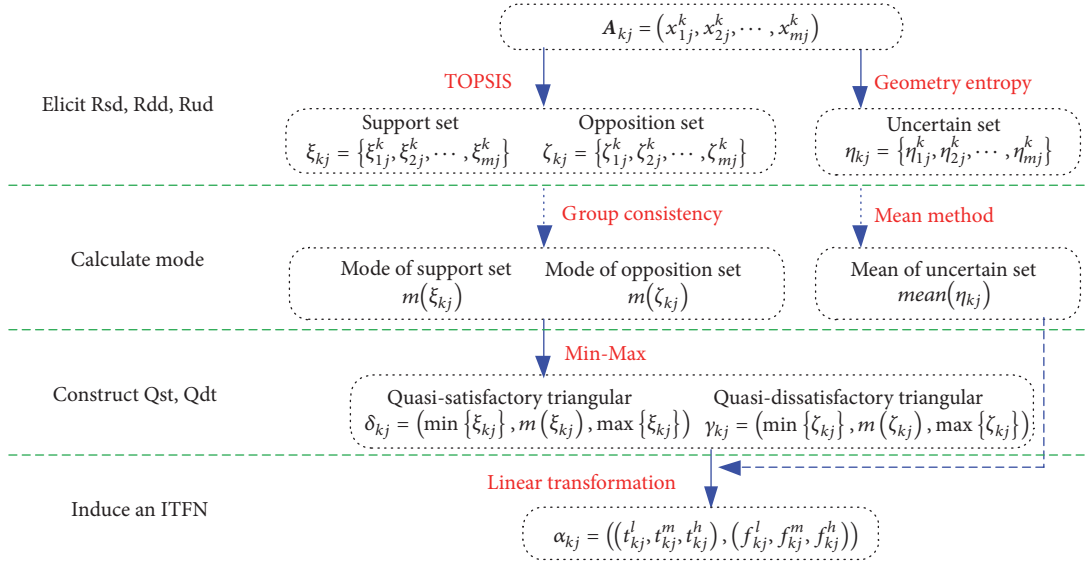


FIGURE 1: Framework for aggregating ITFN.

sets for subsets \widehat{A}_e ($e = 1, 2, 3, 4, 5$) by $N_1 = \{1, 2, \dots, j_1\}$, $N_2 = \{j_1 + 1, j_1 + 2, \dots, j_2\}$, $N_3 = \{j_2 + 1, j_2 + 2, \dots, j_3\}$, $N_4 = \{j_3 + 1, j_3 + 2, \dots, j_4\}$, and $N_5 = \{j_4 + 1, j_4 + 2, \dots, j_5\}$, respectively.

(4) The group decision matrix.

Suppose that the rating of alternative S_k with respect to the attribute A_j given by DM D_i is denoted by x_{ij}^k ($i \in M, j \in N, k \in P$). If $j \in N_1$, then x_{ij}^k is a RN. If $j \in N_2$, then $x_{ij}^k = [\underline{x}_{ij}^k, \bar{x}_{ij}^k]$ is an IN. If $j \in N_3$, then $x_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$ is a TFN. If $j \in N_4$, then $x_{ij}^k = (e_{ij}^k, f_{ij}^k, g_{ij}^k, h_{ij}^k)$ is a TrFN. If $j \in N_5$, then $x_{ij}^k = ((\underline{t}_{ij}^k, \underline{t}_{ij}^k, \bar{t}_{ij}^k), u_{ij}^k, v_{ij}^k)$. Namely, x_{ij}^k can be unified as follows:

$$x_{ij}^k = \begin{cases} x_{ij}^k, & \text{if } j \in N_1 \\ [\underline{x}_{ij}^k, \bar{x}_{ij}^k], & \text{if } j \in N_2 \\ (a_{ij}^k, b_{ij}^k, c_{ij}^k), & \text{if } j \in N_3 \\ (e_{ij}^k, f_{ij}^k, g_{ij}^k, h_{ij}^k), & \text{if } j \in N_4 \\ ((\underline{t}_{ij}^k, \underline{t}_{ij}^k, \bar{t}_{ij}^k), u_{ij}^k, v_{ij}^k), & \text{if } j \in N_5 \end{cases} \quad (16)$$

Hence, a group decision matrix of alternative S_k can be expressed as

$$\mathbf{X}^k = (x_{ij}^k)_{m \times n} = \begin{matrix} & \begin{matrix} A_1 & A_2 & \cdots & A_n \end{matrix} \\ \begin{matrix} D_1 \\ D_2 \\ \vdots \\ D_m \end{matrix} & \begin{pmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{pmatrix} \end{matrix} \quad (17)$$

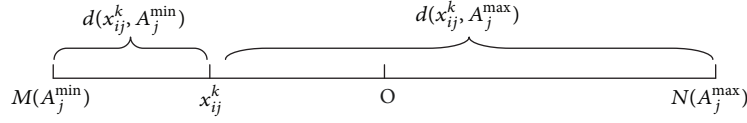
($k \in P$)

To reduce information loss and simplify the focused problems, the group decision matrices $\mathbf{X}^k = (x_{ij}^k)_{m \times n}$ ($k = 1, 2, \dots, p$) can be integrated into a collective ITFN decision matrix. The key to addressing this issue lies in an effective approach for constructing ITFNs based on the experts' assessment expressed in different types of data.

3.2. An Approach to Aggregating Heterogeneous Information into ITFNs. To facilitate the calculation, denote the j th column vector in the matrix \mathbf{X}^k as

$$\mathbf{A}_{kj} = (x_{1j}^k, x_{2j}^k, \dots, x_{mj}^k) \quad (k \in P, j \in N), \quad (18)$$

which is the normalized assessment vector of alternative S_k on attribute A_j given by all DMs D_i ($i = 1, 2, \dots, m$). Let A_j^{\max} and A_j^{\min} be the largest grade and smallest grade employed in the rating system. For example, if the assessments in A_j are TFNs, then $A_j^{\max} = (1, 1, 1)$ and $A_j^{\min} = (0, 0, 0)$; if the assessments in A_j are INs, then $A_j^{\max} = [1, 1]$ and $A_j^{\min} = [0, 0]$. To integrate the decision matrices $\mathbf{X}^k = (x_{ij}^k)_{m \times n}$ ($k = 1, 2, \dots, p$) into a collective ITFN decision matrix, all the elements in vector \mathbf{A}_{kj} need to be aggregated into an ITFN. The implementation of the aggregation approach involves a four-stage framework (see Figure 1): (1) Elicit Rsd, Rdd, and Rud. In this process, we use the TOPSIS method to obtain the rating satisfactory degree (Rsd) and rating dissatisfactory degree (Rdd) of x_{ij}^k and construct the support set ξ_{kj} and opposition set ζ_{kj} of \mathbf{A}_{kj} . The rating uncertain degree (Rud) of x_{ij}^k and the corresponding uncertain set η_{kj} are derived by geometry entropy. (2) Calculate mode. Combining the group consistency and mean method, the modes of the above sets ξ_{kj} , ζ_{kj} , and η_{kj} are computed in this stage. (3) Construct Qst and Qdt. According to the Min-Max method, the quasi-satisfactory triangular (Qst) and quasi-dissatisfactory triangular (Qdt) of \mathbf{A}_{kj} can be built. (4) Induce an ITFN. The ITFN

FIGURE 2: Distances ratio-based rating uncertain degree for x_{ij}^k .

of A_{kj} can be obtained through a linear transformation in this process.

3.2.1. Elicit the Rsd, Rdd, and Rud. Consider that (1) the relative closeness [45] from x_{ij}^k to A_j^{\max} implies the satisfaction of DM; (2) the relative closeness from x_{ij}^k to A_j^{\min} implies the dissatisfaction of DM; and (3) according to the ratio-based measure of fuzziness [46, 47], the ratio of distances from x_{ij}^k to A_j^{\min} and from x_{ij}^k to A_j^{\max} can also express the fuzziness degree of x_{ij}^k . Thus, combining the relative closeness of TOPSIS [36] and geometry entropy method [46], the Rsd, Rdd, and Rud of x_{ij}^k can be elicited as follows.

Definition 7. Let A_{kj} be a benefit attribute vector, and let x_{ij}^k be an arbitrary element in A_{kj} . The Rsd, Rdd, and Rud of x_{ij}^k are defined as

$$\xi_{ij}^k = \frac{d(x_{ij}^k, A_j^{\min})}{d(x_{ij}^k, A_j^{\max}) + d(x_{ij}^k, A_j^{\min})}, \quad (19)$$

$$\zeta_{ij}^k = \frac{d(x_{ij}^k, A_j^{\max})}{d(x_{ij}^k, A_j^{\max}) + d(x_{ij}^k, A_j^{\min})}, \quad (20)$$

$$\eta_{ij}^k = \min \left\{ \frac{d(x_{ij}^k, A_j^{\min})}{d(x_{ij}^k, A_j^{\max})}, \frac{d(x_{ij}^k, A_j^{\max})}{d(x_{ij}^k, A_j^{\min})} \right\}, \quad (21)$$

respectively, where ξ_{ij}^k , ζ_{ij}^k , and η_{ij}^k denote the Rsd, Rdd, and Rud given by d_i on attribute A_j in the alternative S_k , $d(x_{ij}^k, A_j^{\max})$ is the distance between x_{ij}^k and the largest grade A_j^{\max} of the attribute A_j , and $d(x_{ij}^k, A_j^{\min})$ is the distance between x_{ij}^k and the smallest grade A_j^{\min} of the attribute A_j .

For Example 1. Consider that $x_{11}^1 = (4.6, 7.2, 8.3)$ is a TFN in the ten-mark system; then $A_j^{\min} = (0, 0, 0)$, $A_j^{\max} = (10, 10, 10)$. According to (19)-(21), the Rsd, Rdd, and Rud of x_{11}^1 are calculated, respectively, as $\xi_{11}^1 = 0.67$, $\zeta_{11}^1 = 0.33$, and $\eta_{11}^1 = 0.49$.

Theorem 8. Rud η_{ij}^k of x_{ij}^k in A_{kj} has the following properties.

(EP1) If $x_{ij}^k = A_j^{\max}$ or $x_{ij}^k = A_j^{\min}$, then $\eta_{ij}^k = 0$, which means that A_j^{\min} and A_j^{\max} are not fuzzy since A_j^{\min} and A_j^{\max} are crisp sets.

(EP2) If $d(x_{ij}^k, A_j^{\max}) = d(x_{ij}^k, A_j^{\min})$ (namely, x_{ij}^k is the middle point), then $\eta_{ij}^k = 1$, which means that x_{ij}^k is the fuzziest element.

(EP3) $\eta_{ij}^k \leq \eta_{tj}^k$ ($i, t \in M = \{1, 2, \dots, m\}$), if η_{ij}^k is less fuzzy than η_{tj}^k , i.e.,

$$\frac{d(x_{ij}^k, A_j^{\min})}{d(x_{ij}^k, A_j^{\max})} \leq \frac{d(x_{tj}^k, A_j^{\min})}{d(x_{tj}^k, A_j^{\max})},$$

$$\text{for } d(x_{ij}^k, A_j^{\min}) \leq d(x_{tj}^k, A_j^{\min}), d(x_{ij}^k, A_j^{\max}) \leq d(x_{tj}^k, A_j^{\max});$$

$$\frac{d(x_{ij}^k, A_j^{\max})}{d(x_{ij}^k, A_j^{\min})} \leq \frac{d(x_{tj}^k, A_j^{\max})}{d(x_{tj}^k, A_j^{\min})},$$

$$\text{for } d(x_{ij}^k, A_j^{\min}) \geq d(x_{tj}^k, A_j^{\min}), d(x_{ij}^k, A_j^{\max}) \geq d(x_{tj}^k, A_j^{\max});$$

(22)

$$\frac{d(x_{ij}^k, A_j^{\min})}{d(x_{ij}^k, A_j^{\max})} \leq \frac{d(x_{tj}^k, A_j^{\min})}{d(x_{tj}^k, A_j^{\max})},$$

$$\text{for } d(x_{ij}^k, A_j^{\min}) \leq d(x_{tj}^k, A_j^{\min}), d(x_{ij}^k, A_j^{\max}) \geq d(x_{tj}^k, A_j^{\max});$$

$$\frac{d(x_{ij}^k, A_j^{\max})}{d(x_{ij}^k, A_j^{\min})} \leq \frac{d(x_{tj}^k, A_j^{\max})}{d(x_{tj}^k, A_j^{\min})},$$

$$\text{for } d(x_{ij}^k, A_j^{\min}) \geq d(x_{tj}^k, A_j^{\min}), d(x_{ij}^k, A_j^{\max}) \geq d(x_{tj}^k, A_j^{\max}).$$

It is easy to prove that Rud η_{ij}^k meets the properties (EP1)-(EP3), which are consistent with the axioms (1)-(3) of fuzzy entropy based on distance in [48]. From a geometric standpoint, A_j^{\max} and A_j^{\min} in the rating system are nonfuzzy which can correspond to the position M and position N (Figure 2). When a fuzzy number x_{ij}^k is moved from position M (or N) towards middle position O, the distance from x_{ij}^k to M is close to N. Meanwhile, x_{ij}^k become more and more vague; i.e., η_{ij}^k is getting bigger and bigger. Particularly, when x_{ij}^k is in position P, the distance from x_{ij}^k to M is equal to N. So, the fuzzy number x_{ij}^k is the fuzziest, i.e., $\eta_{ij}^k = 1$. According to the analysis above, it is reasonable that Rud measures the uncertainty of the original assessment. It is worth mentioning that (21) is suitable for different forms of decision data such as INs, TFNs, TIFNs, and TrIFNs.

Remark 9. From (19)-(21), the extraction method in this paper relies on just the largest grade A_j^{\max} and the smallest grade A_j^{\min} of the attribute A_j , while the methods [35, 37] rely on A_j^{\max} and A_j^{\min} as well as the middle grade A_j^{mid} of the attribute A_j . However, it is hard to determine the middle grade for some sets of TIFNs [43] and TrIFNs [49, 50]. Hence, the proposed extraction method is more effective and simple.

Remark 10. When A_{kj} is a cost attribute vector, the Rud of x_{ij}^k can be derived by (21). The Rsd and Rdd of x_{ij}^k can be rewritten as

$$\xi_{ij}^k = \frac{d(x_{ij}^k, A_j^{\max})}{d(x_{ij}^k, A_j^{\max}) + d(x_{ij}^k, A_j^{\min})}, \quad (23)$$

$$\zeta_{ij}^k = \frac{d(x_{ij}^k, A_j^{\min})}{d(x_{ij}^k, A_j^{\max}) + d(x_{ij}^k, A_j^{\min})} \quad (24)$$

Remark 11. For the benefit attribute vector $A_{kj} = (x_{1j}^k, x_{2j}^k, \dots, x_{mj}^k)$, ξ_{ij}^k , ζ_{ij}^k , and η_{ij}^k of each element are composed of support set ξ_{kj} , opposition set ζ_{kj} , and uncertain set η_{kj} of A_{kj} which can be defined as $\xi_{kj} = \{\xi_{1j}^k, \xi_{2j}^k, \dots, \xi_{mj}^k\}$ and $\zeta_{kj} = \{\zeta_{1j}^k, \zeta_{2j}^k, \dots, \zeta_{mj}^k\}$ and $\eta_{kj} = \{\eta_{1j}^k, \eta_{2j}^k, \dots, \eta_{mj}^k\}$, respectively.

3.2.2. Calculate the Mode. For the support set $\xi_{kj} = \{\xi_{1j}^k, \xi_{2j}^k, \dots, \xi_{mj}^k\}$, the mode of the TFN is located in the center around which the Rsd ξ_{ij}^k ($i = 1, 2, \dots, m$) gather. Inspired by the literature [49, 50], the more consistent it is with the rest of ξ_{kj} , the greater the importance of the Rsd ξ_{ij}^k given by DM D_i . That is to say, the weighted average of the collection ξ_{kj} can be regarded as its mode. Here, we utilize the distance between ξ_{ij}^k and ξ_{ej}^k to define the consistency degree of D_i on support set ξ_{kj} to the rest of experts, which can be obtained by

$$CI_{ij}^k = \frac{1}{m-1} \sum_{i=1, i \neq j}^m (1 - d(\xi_{ij}^k, \xi_{ej}^k)), \quad (25)$$

where $d(\cdot)$ is the distance between ξ_{ij}^k and other Rsds in ξ_{kj} . Clearly, $0 \leq S_k \leq 1$.

Generally, an expert's Rsd is more important if he/she is more similar to the group's Rsd. In other words, the larger the value of CI_{ij}^k is, the more important ξ_{kj} is. Thus, the weight of D_i on ξ_{kj} can be obtained by

$$u_{ij}^k = \frac{CI_{ij}^k}{\sum_{i=1}^m CI_{ij}^k} \quad (i = 1, 2, \dots, m). \quad (26)$$

Then the mode of δ_{kj} for support set ξ_{kj} is derived as

$$m(\xi_{kj}) = \sum_{i=1}^m u_{ij}^k \xi_{ij}^k. \quad (27)$$

Similarly, we have the mode $m(\zeta_{kj})$ of opposition set ζ_{kj} .

3.2.3. Construct Qst and Qdt. Note that the membership degree and nonmembership degree of a TIFN are TFNs rather than real numbers. Moreover, ξ_{ij}^k and ζ_{ij}^k are real numbers which are difficult to express the imprecise and vague experts' subjective judgment. By doing this, the TFNs of ξ_{ij}^k and ζ_{ij}^k

are commonly used to represent Qsd and Qdd of A_{kj} since TFN is characterized by a membership function. Thus, it is necessary to construct the Qst and Qdt of A_{kj} . As per the definition of TFN, the corresponding TFNs of ξ_{kj} and ζ_{kj} can be constructed as follows.

Definition 12. For the attribute vector A_{kj} , the Qst δ_{kj} and Qdt γ_{kj} of alternative S_k on attribute A_j are defined as

$$\delta_{kj} = (\min\{\xi_{kj}\}, m(\xi_{kj}), \max\{\xi_{kj}\}), \quad (k \in P, j \in N) \quad (28)$$

$$\gamma_{kj} = (\min\{\zeta_{kj}\}, m(\zeta_{kj}), \max\{\zeta_{kj}\}), \quad (k \in P, j \in N) \quad (29)$$

where the $\min\{\xi_{kj}\}$ and $\max\{\xi_{kj}\}$ are the minimum value and maximum value of the support set ξ_{kj} and $\min\{\zeta_{kj}\}$ and $\max\{\zeta_{kj}\}$ are the minimum value and maximum value of opposition set ζ_{kj} . For the convenience of discussion, the pair $(\delta_{kj}, \gamma_{kj})$ is called a quasi-ITFN.

Remark 13. To calculate the mode of triangular fuzzy numbers δ_{kj} and γ_{kj} , this paper employs the weighted averaging value that considers the distribution of ratings, whereas some works used the mean value method. The essential difference is that the current method takes the consistency of the group into account, while the mean value method is based on statistical assumptions.

3.2.4. Inducing an ITFN. Finally, an ITFN is induced from the Qst and Qdt of alternative S_k on the attribute A_j by the following normalized method. Let $\alpha_{kj} = ((t_{kj}^l, t_{kj}^m, t_{kj}^h), (f_{kj}^l, f_{kj}^m, f_{kj}^h))$ ($k \in P, j \in N$) be the induced ITFN by the attribute vector A_{kj} , and $\text{mean}(\eta_{kj}) = (1/m) \sum_{i=1}^m \eta_{ij}^k$ is the uncertain degree of A_{kj} . To satisfy the conditions in (3) and consider the influence of uncertain degree, the values of $t_{kj}^l, t_{kj}^m, t_{kj}^h, f_{kj}^l, f_{kj}^m, f_{kj}^h$ can be computed as follows:

$$t_{kj}^l = \frac{\min\{\xi_{kj}\}}{\psi_{kj}},$$

$$t_{kj}^m = \frac{\text{mean}(\xi_{kj})}{\psi_{kj}},$$

$$t_{kj}^h = \frac{\max\{\xi_{kj}\}}{\psi_{kj}},$$

$$f_{kj}^l = \frac{\min\{\zeta_{kj}\}}{\psi_{kj}},$$

$$f_{kj}^m = \frac{\text{mean}(\zeta_{kj})}{\psi_{kj}},$$

$$f_{kj}^h = \frac{\max\{\zeta_{kj}\}}{\psi_{kj}}, \quad (k \in P, j \in N) \quad (30)$$

respectively, where $\psi_{kj} = (1/5)(\min\{\xi_{kj}\} + m(\xi_{kj}) + \min\{\zeta_{kj}\} + m(\zeta_{kj}) + \text{mean}(\eta_{kj})) + \max\{\xi_{kj}\} + \max\{\zeta_{kj}\}$.

Apparently, $t_{kj}^l, t_{kj}^m, t_{kj}^h, f_{kj}^l, f_{kj}^m, f_{kj}^h$ satisfy (3). Thus, $\alpha_{kj} = ((t_{kj}^l, t_{kj}^m, t_{kj}^h), (f_{kj}^l, f_{kj}^m, f_{kj}^h))$ is an ITFN. Namely, all the attribute values in the vector A_{kj} can be aggregated into an ITFN α_{kj} .

For Example 2. Consider that $A_{kj} = ((4.6, 7.2, 8.3), (3.4, 4.6, 6.7), (5.6, 6.2, 8.2), (4.5, 6.5, 10), (3.7, 6.5, 7.2))$ is a TFN vector in the ten-mark system; then $A_j^{\min} = (0, 0, 0)$,

$$\begin{matrix} & A_1 & A_2 & \dots & A_n \\ \begin{matrix} S_1 \\ S_2 \\ \vdots \\ S_p \end{matrix} & \left(\begin{matrix} ((t_{11}^l, t_{11}^m, t_{11}^h), (f_{11}^l, f_{11}^m, f_{11}^h)) & ((t_{12}^l, t_{12}^m, t_{12}^h), (f_{12}^l, f_{12}^m, f_{12}^h)) & \dots & ((t_{1n}^l, t_{1n}^m, t_{1n}^h), (f_{1n}^l, f_{1n}^m, f_{1n}^h)) \\ ((t_{21}^l, t_{21}^m, t_{21}^h), (f_{21}^l, f_{21}^m, f_{21}^h)) & ((t_{22}^l, t_{22}^m, t_{22}^h), (f_{22}^l, f_{22}^m, f_{22}^h)) & \dots & ((t_{2n}^l, t_{2n}^m, t_{2n}^h), (f_{2n}^l, f_{2n}^m, f_{2n}^h)) \\ \vdots & \vdots & \vdots & \vdots \\ ((t_{p1}^l, t_{p1}^m, t_{p1}^h), (f_{p1}^l, f_{p1}^m, f_{p1}^h)) & ((t_{p2}^l, t_{p2}^m, t_{p2}^h), (f_{p2}^l, f_{p2}^m, f_{p2}^h)) & \dots & ((t_{pn}^l, t_{pn}^m, t_{pn}^h), (f_{pn}^l, f_{pn}^m, f_{pn}^h)) \end{matrix} \right) \end{matrix} \quad (31)$$

where $r_{kj} = ((t_{kj}^l, t_{kj}^m, t_{kj}^h), (f_{kj}^l, f_{kj}^m, f_{kj}^h))$ ($k \in P, j \in N$) are ITFNs aggregated by the attribute vector A_{kj} in X^k .

Assume that the weight vector $w = (w_1, w_2, \dots, w_n)$ of attributes is fully known; by (4), we can easily obtain the comprehensive rating $r_k = ((t_k^l, t_k^m, t_k^h), (f_k^l, f_k^m, f_k^h))$ of the alternative S_k . When w is incompletely known, we may utilize the following programming model to establish the attribute weights.

4.1. Determination of Attribute Weights. As all known, it is reasonable to determine the attribute weights by making alternative similar to the positive ideal solution (PIS) and at the same time far away from the negative ideal solution (NIS) as far as possible. Let $S^+ = \{r_1^+, r_2^+, \dots, r_n^+\}$ and $S^- = \{r_1^-, r_2^-, \dots, r_n^-\}$ be the intuitionistic triangular fuzzy PIS and NIS, respectively, where

$$\begin{aligned} r_j^+ &= \left(\left(\max_k t_{kj}^l, \max_k t_{kj}^m, \max_k t_{kj}^h \right), \right. \\ &\quad \left(\min_k f_{kj}^l, \min_k f_{kj}^m, \min_k f_{kj}^h \right) \right) \\ &= \left((t_j^{l+}, t_j^{m+}, t_j^{h+}), (f_j^{l-}, f_j^{m-}, f_j^{h-}) \right), \end{aligned} \quad (32)$$

$$\begin{aligned} r_j^- &= \left(\left(\min_k t_{kj}^l, \min_k t_{kj}^m, \min_k t_{kj}^h \right), \right. \\ &\quad \left(\max_k f_{kj}^l, \max_k f_{kj}^m, \max_k f_{kj}^h \right) \right) \\ &= \left((t_j^{l-}, t_j^{m-}, t_j^{h-}), (f_j^{l+}, f_j^{m+}, f_j^{h+}) \right). \end{aligned} \quad (33)$$

$A_j^{\max} = (10, 10, 10)$. According to (19)-(21), the support set, opposition set, and uncertain set of A_{kj} are calculated, respectively, as $\xi_{kj} = \{0.67, 0.49, 0.70, 0.65, 0.58\}$, $\zeta_{kj} = \{0.33, 0.51, 0.30, 0.35, 0.42\}$, and $\eta_{kj} = \{0.49, 0.39, 0.52, 0.5, 0.44\}$. Subsequently, the mode of the sets ξ_{kj} and ζ_{kj} is calculated from (25)-(27) to be $m(\xi_{kj}) = 0.62$ and $m(\zeta_{kj}) = 0.38$. It follows from (28) and (29) that $\delta_{kj} = (0.49, 0.62, 0.70)$ and $\gamma_{kj} = (0.30, 0.38, 0.51)$. Finally, by using (29), the induced ITFN associated with A_{kj} is derived to be $\alpha_{kj} = ((0.349, 0.445, 0.499), (0.214, 0.268, 0.364))$.

4. A Novel Approach for Heterogeneous MAGDM Problems

According to the proposed aggregation method, the collective decision matrix $X = (r_{kj})_{p \times n}$ is aggregated as follows:

($j \in N = \{1, 2, \dots, n\}$). Based on (5), we can define the similarity degree between alternative S_k ($k \in P = \{1, 2, \dots, p\}$) and intuitionistic triangular fuzzy ideal solution, shown as follows:

$$\begin{aligned} \vartheta_k^+ &= 1 - \sum_{j=1}^p w_j * \left[\frac{1}{12} \left(|t_{kj}^l - t_j^{l+}| + |t_{kj}^m - t_j^{m+}| + |t_{kj}^h - t_j^{h+}| \right. \right. \\ &\quad \left. \left. - t_j^{h+} + |f_{kj}^l - f_j^{l+}| + |f_{kj}^m - f_j^{m+}| + |f_{kj}^h - f_j^{h+}| \right) \right. \\ &\quad \left. + \frac{1}{2} \max \left(|t_{kj}^l - t_j^{l+}|, |t_{kj}^m - t_j^{m+}|, |t_{kj}^h - t_j^{h+}|, \right. \right. \\ &\quad \left. \left. |f_{kj}^l - f_j^{l+}|, |f_{kj}^m - f_j^{m+}|, |f_{kj}^h - f_j^{h+}| \right) \right] \end{aligned} \quad (34)$$

$$\begin{aligned} \vartheta_k^- &= 1 - \sum_{j=1}^p w_j * \left[\frac{1}{12} \left(|t_{kj}^l - t_j^{l-}| + |t_{kj}^m - t_j^{m-}| + |t_{kj}^h - t_j^{h-}| \right. \right. \\ &\quad \left. \left. - t_j^{h-} + |f_{kj}^l - f_j^{l-}| + |f_{kj}^m - f_j^{m-}| + |f_{kj}^h - f_j^{h-}| \right) \right. \\ &\quad \left. + \frac{1}{2} \max \left(|t_{kj}^l - t_j^{l-}|, |t_{kj}^m - t_j^{m-}|, |t_{kj}^h - t_j^{h-}|, \right. \right. \\ &\quad \left. \left. |f_{kj}^l - f_j^{l-}|, |f_{kj}^m - f_j^{m-}|, |f_{kj}^h - f_j^{h-}| \right) \right] \end{aligned} \quad (35)$$

Then, a multiple objective linear mathematical programming model is constructed as follows:

$$\begin{aligned} \max \quad & \{\vartheta_k^+ - \vartheta_k^-\} \quad (k \in P) \\ \text{s.t.} \quad & w \in \Omega \end{aligned} \quad (36)$$

For a multiple objective programming, there are several solution methods. Here, we apply the Max-Min method. Let $\rho = \min\{\vartheta_k^+ - \vartheta_k^-\}$. By using the Max-Min method, (36) can be solved by the following single objective linear programming model:

$$\begin{aligned} \max \quad & \rho \\ \text{s.t.} \quad & \begin{cases} \vartheta_k^+ - \vartheta_k^- \geq \rho & (k \in P) \\ \mathbf{w} \in \Omega \end{cases} \end{aligned} \quad (37)$$

By plugging (34) and (35) in (37), it is easily seen that we can derive the attribute weights since the optimal solution of (37) is a Pareto optimal solution of (36).

Remark 14. To obtain the weights of attributes in the intuitionistic triangular fuzzy environment, the current methodology first combines a multiple objective mathematical programming model with TOPSIS idea based on the above collective decision matrix. Then, this model can be solved by Max-Min method, which is relatively simple. However, Li and Chen [49] determined the attribute weights by expected weight value that involved options of decision makers. Shan and Xu [42] gave the attribute weights in advance. Therefore, our method could be more reasonable and objective.

Thus, the ranking order of the alternative S_k can be conducted by the following relative closeness coefficient (RCC):

$$RCC_k = \frac{\vartheta_k^+}{\vartheta_k^+ + \vartheta_k^-} \quad (38)$$

where $0 \leq RCC_k \leq 1$, $(k \in P)$. It is obvious that the larger RCC_k , the better the alternative S_k .

4.2. Procedure of the Proposed Aggregation Method for Heterogeneous MAGDM. On the basis of the above analysis, a new approach to heterogeneous MAGDM problems involves the following primary steps.

Step 1. Obtain the decision matrix of alternative S_k ($k = 1, 2, \dots, p$) taking the form of $\mathbf{Y}^k = (y_{ij}^k)_{m \times n}$, by (18).

Step 2. Convert the decision matrices $\mathbf{X}^k = (x_{ij}^k)_{m \times n}$ ($k = 1, 2, \dots, p$) into a collective decision matrix $\mathbf{X} = (r_{kj})_{p \times n}$ through the aggregation method developed in Section 3. Based on each column vector \mathbf{A}_{kj} in the matrix \mathbf{X}^k , conduct the following substeps.

Step 2.1. Compute the Rsd, Rdd, and Rud of x_{ij}^k by plugging the Hamming distance of different attribute types into (19), (20), (21), (23), and (24), and construct support set ξ_{kj} , opposition set ζ_{kj} , and uncertain set η_{kj} of \mathbf{A}_{kj} .

Step 2.2. Calculate the mode of the support set ξ_{kj} and opposition set ζ_{kj} of \mathbf{A}_{kj} by (25)-(27).

Step 2.3. Construct the Qst δ_{kj} and Qdt γ_{kj} of \mathbf{A}_{kj} by (28) and (29).

Step 2.4. Induce the corresponding ITFN of \mathbf{A}_{kj} using (30).

Step 3. Determine the attribute weights by constructing a multiple objective programming model. The detailed steps are as follows.

Step 3.1. Define the intuitionistic triangular fuzzy PIS $S^+ = \{r_1^+, r_2^+, \dots, r_n^+\}$ and NIS $S^- = \{r_1^-, r_2^-, \dots, r_n^-\}$ by (32) and (33), respectively.

Step 3.2. Compute the similarity degrees ϑ_k^+ and ϑ_k^- from the elements at the k th row of the collective decision matrix $\mathbf{X} = (r_{kj})_{p \times n}$ to PIS S^+ and NIS S^- by (34) and (35), respectively.

Step 3.3. Construct a multiple objective programming model based on (36).

Step 3.4. Convert the above model into a single objective programming model by (37).

Step 3.5. Obtain the optimal weights of attributes by solving the linear programming model.

Step 4. Calculate the similarity degrees ϑ_k^+ and ϑ_k^- of alternative S_k ($k \in P$) by the obtained attribute weights and (34) and (35).

Step 5. Calculate the RCC of alternative S_k ($k \in P$) by (38).

Step 6. Rank the alternatives according to the RCC and select the best one.

The decision procedure of the proposed algorithm may be depicted in Figure 3.

5. A Trustworthy Seller Selection Problem and Comparison Analyses

To demonstrate the efficacy of the proposed HMAGDM method, this section gives a trustworthy seller selection example and conducts comparison analyses with the ones of the existing methods [34, 35, 37].

5.1. A Trustworthy Seller Selection Example and Its Solution Procedure. Online service trading usually takes place between parties who are autonomous, in an environment where the buyer often has not enough information about the seller and goods. Many scholars think that trust is a prerequisite for successful trading. Therefore, it is very important that buyers can identify the most trustworthy seller. Suppose that a consumer desires to select a trustworthy seller. After preliminary screening, four candidate sellers S_1 , S_2 , S_3 , and S_4 remain to be further evaluated. Based on detailed seller ratings, the decision-making committee assesses the

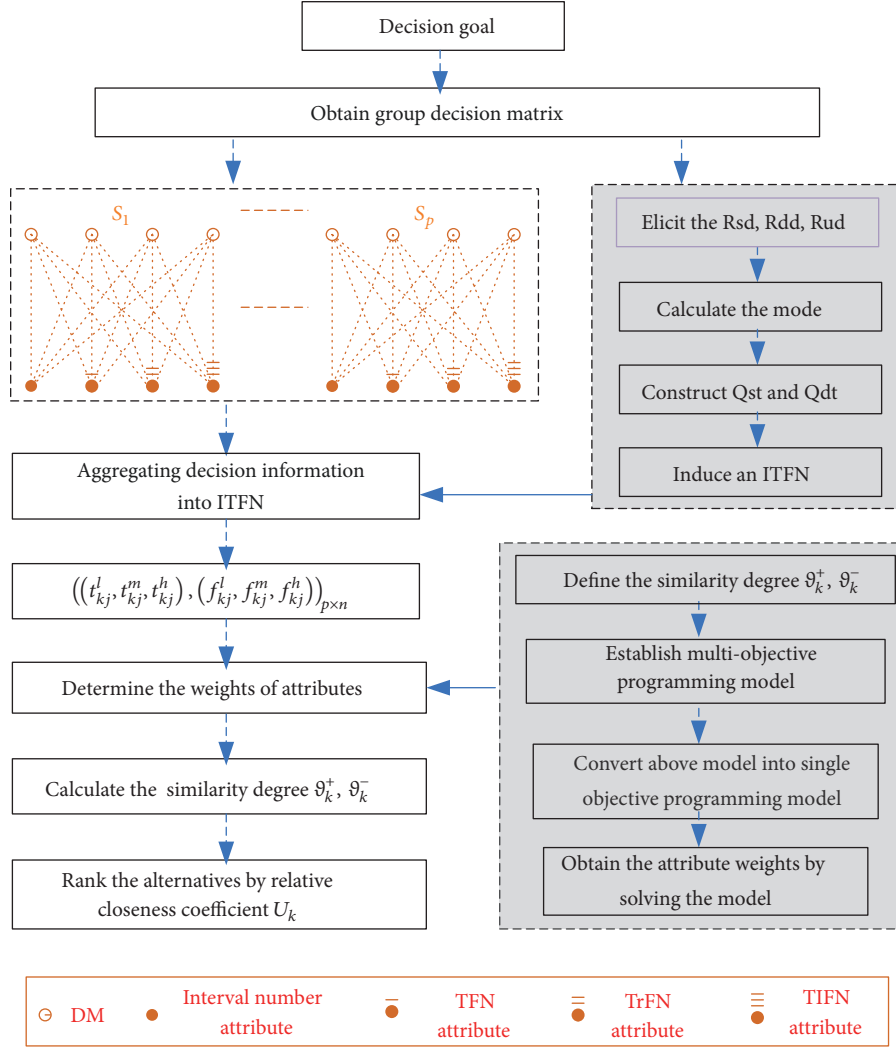


FIGURE 3: The framework of the proposed heterogeneous MAGDM method.

four candidate sellers according to the five trust factors, including product quality (A_1), service attitude (A_2), website usability (A_3), response time (A_4), and shipping speed (A_5).

Product quality (A_1), service attitude (A_2), and website usability (A_3) given by DMs with a one-mark system are all benefit attributes. It is better to use TFNs to assess product quality (A_1). For service attitude (A_2), the experts like to provide the lower and upper limits and the most possible intervals; thus the assessments of A_2 can be represented by

TrFNs. The website usability (A_3) is expressed by TIFNs, while response time (A_4) and shipping speed (A_5) given by DMs with a ten-mark system both are cost attributes. The assessments of the sellers on A_4 can be represented by RNs. Due to the uncertainty of shipping speed, INs are suitably utilized to represent the assessments of shipping speed (A_5). The assessments of four sellers on five attributes given by five experts are listed in Table 1. The attributes' importance is incomplete and experts give incomplete information on the attributes' importance as follows:

$$\Omega = \begin{cases} w_1 \geq 0.1; w_2 - w_1 \leq 0.05; w_3 - w_1 \leq 0.05; w_5 - w_1 \leq 0.05; \\ w_1 + w_3 + w_5 \leq 0.6; w_4 \leq 0.2; w_1 + w_2 + w_3 + w_4 + w_5 = 1. \end{cases} \quad (39)$$

Obviously, the decision problem mentioned above is a heterogeneous MAGDM problem involving five different formats of data: RNs, INs, TFNs, TrFNs, and TIFNs.

To address this problem, we apply the proposed decision method to the selection of the trustworthy sellers below.

TABLE 1: The decision matrix of four alternatives.

Sellers	DMs	A_1	A_2	A_3	A_4	A_5
S_1	D_1	(0.46,0.72,0.83)	(0.2, 0.3, 0.5, 1.0)	((0.33,0.5,0.67),0.5,0.3)	3.4	[2, 6]
	D_2	(0.34,0.46,0.67)	(0.4, 0.5, 0.6, 0.8)	((0.33,0.5,0.67),0.4,0.5)	5.4	[1, 3]
	D_3	(0.56,0.62,0.82)	(0.4, 0.4, 0.5, 0.8)	((0.33,0.5,0.67),0.6,0.1)	7.2	[1, 6]
	D_4	(0.45,0.65,1.00)	(0.2, 0.3, 0.5, 0.9)	((0.5,0.67,0.83),0.7,0.2)	8.1	[1, 4]
	D_5	(0.37,0.65,0.72)	(0.0, 0.1, 0.3, 0.7)	((0.17,0.33,0.5),0.4,0.6)	5.9	[3, 8]
S_2	D_1	(0.51,0.67,0.73)	(0.2, 0.3, 0.4, 0.7)	((0.67,0.83,1.0),0.6,0.3)	4.2	[3, 6]
	D_2	(0.35,0.64,0.73)	(0.3, 0.4, 0.4, 0.9)	((0.33,0.5,0.67),0.4,0.5)	5.4	[2, 5]
	D_3	(0.66,0.82,1.00)	(0.2, 0.2, 0.4, 0.5)	((0.67,0.83,1.0),0.6,0.2)	7.2	[4, 7]
	D_4	(0.34,0.51,0.77)	(0.5, 0.6, 0.8, 0.8)	((0.17,0.33,0.5),0.7,0.2)	10.0	[1, 3]
	D_5	(0.68,0.82,1.00)	(0.1, 0.1, 0.2, 0.5)	((0.67,0.83,1.0),0.3,0.6)	7.1	[6, 7]
S_3	D_1	(0.52,0.66,0.81)	(0.5, 0.6, 0.6, 0.8)	((0.0,0.17,0.33),0.5,0.3)	7.8	[2, 4]
	D_2	(0.39,0.66,0.68)	(0.4, 0.5, 0.7, 0.8)	((0.33,0.5,0.67),0.4,0.5)	6.5	[2, 4]
	D_3	(0.62,0.67,0.84)	(0.3, 0.3, 0.5, 0.9)	((0.0,0.17,0.33),0.6,0.1)	4.3	[1, 3]
	D_4	(0.58,0.74,1.00)	(0.4, 0.4, 0.6, 0.8)	((0.33,0.5,0.67),0.6,0.3)	8.3	[3, 5]
	D_5	(0.44,0.50,0.67)	(0.3, 0.3, 0.4, 0.5)	((0.5,0.67,0.83),0.4,0.6)	5.5	[4, 7]
S_4	D_1	(0.34,0.47,0.67)	(0.2, 0.3, 0.4, 0.5)	((0.5,0.67,0.83),0.4,0.4)	4.9	[6, 7]
	D_2	(0.59,0.66,0.68)	(0.2, 0.3, 0.4, 0.4)	((0.5,0.67,0.83),0.6,0.4)	8.1	[5, 6]
	D_3	(0.47,0.56,0.67)	(0.1, 0.1, 0.2, 0.7)	((0.33,0.5,0.67),0.6,0.1)	3.9	[1, 10]
	D_4	(0.45,0.50,0.69)	(0.4, 0.5, 0.7, 0.8)	((0.33,0.5,0.67),0.7,0.3)	5.8	[2, 4]
	D_5	(0.57,0.74,1.00)	(0.2, 0.3, 0.6, 0.6)	((0.17,0.33,0.5),0.4,0.6)	7.0	[4, 6]

Step 1. The group decision matrices are obtained as in Table 1.

Step 2. Due to the ratings of A_1, A_2, A_3 given by DMs based on the one-mark system and the ratings of A_4, A_5 , with the ten-mark system, we have

$$\begin{aligned}
 A_j^{\min} &= \begin{cases} 0, & \text{if } j \in N_1 \\ [0, 0], & \text{if } j \in N_2 \\ (0, 0, 0), & \text{if } j \in N_3 \\ (0, 0, 0, 0), & \text{if } j \in N_4 \\ ((0, 0, 0), 0, 1), & \text{if } j \in N_5, \end{cases} \\
 A_j^{\max} &= \begin{cases} 10, & \text{if } j \in N_1 \\ [10, 10], & \text{if } j \in N_2 \\ (1, 1, 1), & \text{if } j \in N_3 \\ (1, 1, 1, 1), & \text{if } j \in N_4 \\ ((1, 1, 1), 1, 0), & \text{if } j \in N_5. \end{cases} \quad (40)
 \end{aligned}$$

Obtain the aggregated ITFNs corresponding to the attribute vectors.

Step 2.1. By plugging (15) into (19)-(21) for benefit attributes (plugging (15) into (23), (24), and (21) for cost attributes), we can compute the Rsd, Rdd, and Rud of x_{ij}^k , which are shown in Table 2. Thereby the support set ξ_{kj} and opposition set ζ_{kj} of A_{kj} can be derived.

Step 2.2. By (25)-(27), we can obtain the modes of ξ_{kj} and ζ_{kj} , which are listed in Table 3.

Step 2.3. Using (28) and (29), the Qst δ_{kj} and Qdt γ_{kj} of A_{kj} can be constructed, and the results are presented in Table 3.

Step 2.4. Based on (30), the aggregated ITFNs of A_{kj} are also shown in Table 3.

Step 3. Determine the attribute weights.

Step 3.1. By (32) and (33), the intuitionistic triangular fuzzy PIS and NIS are defined as follows.

$$\begin{aligned}
 S^+ &= \{((0.41, 0.51, 0.63), (0.13, 0.24, 0.34)), \\
 &\quad ((0.23, 0.33, 0.39), (0.18, 0.29, 0.39)), \\
 &\quad ((0.28, 0.38, 0.46), (0.19, 0.27, 0.36)), \\
 &\quad ((0.10, 0.22, 0.36), (0.18, 0.32, 0.44)), \\
 &\quad ((0.32, 0.46, 0.56), (0.13, 0.24, 0.35))\}, \\
 S^- &= \{((0.35, 0.42, 0.50), (0.21, 0.28, 0.36)), \\
 &\quad ((0.13, 0.22, 0.33), (0.24, 0.34, 0.43)), \\
 &\quad ((0.07, 0.18, 0.29), (0.24, 0.34, 0.45)), \\
 &\quad ((0.00, 0.16, 0.29), (0.23, 0.34, 0.49)), \\
 &\quad ((0.22, 0.32, 0.44), (0.19, 0.31, 0.41))\}. \quad (41)
 \end{aligned}$$

Step 3.2. By (34) and (35), the similarity degrees ϑ_k^+ and ϑ_k^- ($k = 1, 2, 3, 4$) are calculated as follows.

$$\begin{aligned}
 \vartheta_1^+ &= 1 - (0.097w_1 + 0.062w_2 + 0.051w_3 + 0.004w_4 \\
 &\quad + 0.026w_5)
 \end{aligned}$$

TABLE 2: Rsd, Rdd and Rud of each attribute value.

Providers	DM	A_1	A_2	A_3	A_4	A_5
S_1	D_1	0.67,0.33,0.49	0.50,0.50,1.00	0.42,0.58,0.64	0.66,0.34,0.52	0.60,0.40,0.67
	D_2	0.49,0.51,0.39	0.58,0.42,0.74	0.42,0.58,0.61	0.46,0.54,0.85	0.80,0.20,0.25
	D_3	0.67,0.33,0.52	0.53,0.47,0.90	0.41,0.59,0.68	0.28,0.72,0.39	0.65,0.35,0.54
	D_4	0.70,0.30,0.50	0.48,0.52,0.90	0.59,0.41,0.79	0.19,0.81,0.24	0.75,0.25,0.33
	D_5	0.58,0.42,0.44	0.28,0.72,0.38	0.67,0.33,0.80	0.41,0.59,0.70	0.45,0.55,0.82
S_2	D_1	0.64,0.36,0.49	0.40,0.60,0.67	0.70,0.30,0.58	0.58,0.42,0.72	0.55,0.45,0.82
	D_2	0.57,0.43,0.43	0.50,0.50,1.00	0.42,0.58,0.61	0.46,0.54,0.85	0.65,0.35,0.54
	D_3	0.82,0.18,0.60	0.33,0.67,0.68	0.69,0.31,0.56	0.28,0.72,0.39	0.45,0.55,0.82
	D_4	0.54,0.46,0.41	0.68,0.32,0.33	0.45,0.55,0.76	0.00,1.00,0.00	0.80,0.20,0.25
	D_5	0.83,0.17,0.60	0.23,0.77,0.78	0.63,0.37,0.89	0.29,0.71,0.41	0.35,0.65,0.54
S_3	D_1	0.66,0.34,0.50	0.63,0.37,0.60	0.15,0.85,0.17	0.22,0.78,0.28	0.70,0.30,0.43
	D_2	0.58,0.42,0.44	0.60,0.40,0.67	0.42,0.58,0.61	0.35,0.65,0.54	0.70,0.30,0.43
	D_3	0.71,0.29,0.55	0.50,0.50,1.00	0.14,0.86,0.16	0.57,0.43,0.75	0.80,0.20,0.25
	D_4	0.77,0.23,0.56	0.55,0.45,0.82	0.44,0.56,0.70	0.17,0.83,0.21	0.60,0.40,0.67
	D_5	0.54,0.46,0.43	0.38,0.62,0.60	0.56,0.44,0.91	0.45,0.55,0.82	0.45,0.55,0.82
S_4	D_1	0.49,0.51,0.39	0.35,0.65,0.54	0.53,0.47,0.90	0.51,0.49,0.96	0.45,0.55,0.82
	D_2	0.64,0.36,0.51	0.33,0.67,0.48	0.59,0.41,0.90	0.19,0.81,0.24	0.45,0.55,0.82
	D_3	0.57,0.43,0.45	0.28,0.72,0.38	0.41,0.59,0.68	0.61,0.39,0.64	0.45,0.55,0.82
	D_4	0.55,0.45,0.44	0.60,0.40,0.67	0.47,0.53,0.77	0.42,0.58,0.72	0.70,0.30,0.43
	D_5	0.77,0.23,0.55	0.43,0.57,0.74	0.56,0.44,0.91	0.30,0.70,0.43	0.50,0.50,1.00

$$\vartheta_2^+ = 1 - (0.002w_1 + 0.078w_2 + 0.008w_3 + 0.078w_4 + 0.086w_5)$$

$$\vartheta_3^+ = 1 - (0.040w_1 + 0.030w_2 + 0.165w_3 + 0.043w_4 + 0.026w_5)$$

$$\vartheta_4^+ = 1 - (0.072w_1 + 0.085w_2 + 0.098w_3 + 0.018w_4 + 0.109w_5)$$

$$\vartheta_1^- = 1 - (0.014w_1 + 0.038w_2 + 0.141w_3 + 0.079w_4 + 0.114w_5)$$

$$\vartheta_2^- = 1 - (0.100w_1 + 0.043w_2 + 0.165w_3 + 0.146w_4 + 0.267w_5)$$

$$\vartheta_3^- = 1 - (0.060w_1 + 0.087w_2 + 0.003w_3 + 0.166w_4 + 0.323w_5)$$

$$\vartheta_4^- = 1 - (0.034w_1 + 0.021w_2 + 0.077w_3 + 0.191w_4 + 0.230w_5)$$

(42)

Step 3.3. By using (36), a multiple objective programming model is expressed as follows:

max

$$\{\vartheta_1^+ - \vartheta_1^- = -0.084w_1 - 0.023w_2 + 0.089w_3 + 0.075w_4 + 0.088w_5\}$$

max

$$\{\vartheta_2^+ - \vartheta_2^- = 0.098w_1 - 0.035w_2 + 0.156w_3 + 0.068w_4 + 0.181w_5\}$$

max

$$\{\vartheta_3^+ - \vartheta_3^- = 0.020w_1 + 0.058w_2 - 0.162w_3 + 0.123w_4 + 0.298w_5\}$$

max

$$\{\vartheta_4^+ - \vartheta_4^- = -0.038w_1 - 0.064w_2 - 0.021w_3 + 0.172w_4 + 0.122w_5\}$$

s.t. $\mathbf{w} \in \Omega$

(43)

TABLE 3: Mode $m\{\xi_{kj}\}$, $m\{\zeta_{kj}\}$, Qst δ_{kj} , Qdt γ_{kj} and aggregated ITFNs of attribute vectors.

Sellers	Attributes	$m\{\xi_{kj}\}$	ξ_{kj}	$m\{\zeta_{kj}\}$	ζ_{kj}	ITFNs
S_1	A_1	0.62	(0.49,0.62,0.70)	0.38	(0.30,0.38,0.51)	((0.35,0.44,0.49),(0.21,0.27,0.36))
	A_2	0.48	(0.28,0.48,0.58)	0.52	(0.43,0.52,0.73)	((0.16,0.28,0.34),(0.25,0.31,0.43))
	A_3	0.59	(0.50,0.59,0.83)	0.41	(0.17,0.41,0.50)	((0.35,0.41,0.58),(0.12,0.29,0.35))
	A_4	0.40	(0.19,0.40,0.66)	0.60	(0.34,0.60,0.81)	((0.10,0.22,0.36),(0.19,0.33,0.45))
	A_5	0.65	(0.45,0.65,0.80)	0.35	(0.20,0.35,0.55)	((0.32,0.46,0.56),(0.14,0.24,0.39))
S_2	A_1	0.68	(0.54,0.68,0.83)	0.32	(0.17,0.32,0.46)	((0.40,0.51,0.62),(0.12,0.24,0.34))
	A_2	0.42	(0.23,0.42,0.68)	0.58	(0.33,0.58,0.78)	((0.13,0.24,0.38),(0.18,0.33,0.44))
	A_3	0.71	(0.50,0.71,0.83)	0.29	(0.17,0.29,0.50)	((0.37,0.53,0.62),(0.12,0.22,0.37))
	A_4	0.67	(0.00,0.33,0.58)	0.67	(0.42,0.67,1.00)	((0.00,0.16,0.29),(0.21,0.33,0.50))
	A_5	0.44	(0.35,0.56,0.80)	0.44	(0.20,0.44,0.65)	((0.22,0.35,0.51),(0.13,0.28,0.41))
S_3	A_1	0.65	(0.54,0.65,0.77)	0.35	(0.23,0.35,0.46)	((0.40,0.48,0.57),(0.17,0.26,0.34))
	A_2	0.53	(0.38,0.53,0.63)	0.47	(0.38,0.47,0.63)	((0.24,0.34,0.40),(0.24,0.30,0.40))
	A_3	0.40	(0.17,0.40,0.67)	0.60	(0.33,0.60,0.83)	((0.09,0.22,0.37),(0.18,0.33,0.46))
	A_4	0.35	(0.17,0.35,0.57)	0.65	(0.43,0.65,0.83)	((0.09,0.19,0.31),(0.23,0.35,0.45))
	A_5	0.46	(0.32,0.46,0.56)	0.24	(0.14,0.24,0.39)	((0.32,0.46,0.56),(0.14,0.24,0.39))
S_4	A_1	0.60	(0.49,0.60,0.77)	0.40	(0.23,0.40,0.51)	((0.35,0.42,0.54),(0.16,0.28,0.35))
	A_2	0.39	(0.28,0.39,0.60)	0.61	(0.40,0.61,0.73)	((0.16,0.22,0.34),(0.23,0.35,0.41))
	A_3	0.54	(0.33,0.54,0.67)	0.46	(0.33,0.46,0.67)	((0.21,0.34,0.42),(0.21,0.29,0.42))
	A_4	0.41	(0.19,0.41,0.61)	0.59	(0.39,0.59,0.81)	((0.11,0.23,0.34),(0.22,0.33,0.45))
	A_5	0.50	(0.45,0.50,0.70)	0.40	(0.30,0.40,0.55)	((0.29,0.33,0.46),(0.20,0.32,0.36))

Step 3.4. By using (37), (43) can be changed into the following single objective linear programming model:

$$\begin{aligned}
 & \max \quad \{x\} \\
 & \text{s.t.} \quad \begin{cases} -0.084w_1 - 0.020w_2 + 0.089w_3 + 0.075w_4 + 0.088w_5 \geq x \\ 0.098w_1 - 0.030w_2 + 0.154w_3 + 0.068w_4 + 0.181w_5 \geq x \\ 0.020w_1 + 0.058w_2 - 0.162w_3 + 0.123w_4 + 0.298w_5 \geq x \\ -0.038w_1 - 0.070w_2 - 0.021w_3 + 0.172w_4 + 0.122w_5 \geq x \\ \mathbf{w} \in \Omega \end{cases} \quad (44)
 \end{aligned}$$

Step 3.5. Applying Lingo 11.0, we have the attribute weight vector $\mathbf{w} = (0.162, 0.212, 0.214, 0.200, 0.212)$.

Step 4. By plugging \mathbf{w} into (34) and (35), the similarity degrees ϑ_k^+ and ϑ_k^- of alternative S_k can be calculated as follows:

$$\begin{aligned}
 \vartheta_1^+ &= 0.954, \\
 \vartheta_1^- &= 0.919, \\
 \vartheta_2^+ &= 0.948, \\
 \vartheta_2^- &= 0.854, \\
 \vartheta_3^+ &= 0.938,
 \end{aligned}$$

$$\vartheta_3^- = 0.869,$$

$$\vartheta_4^+ = 0.923,$$

$$\vartheta_4^- = 0.886.$$

(45)

Step 5. Based on (37), the RCC of alternative S_k is yielded as follows:

$$\begin{aligned}
 RCC_1 &= 0.509, \\
 RCC_2 &= 0.526, \\
 RCC_3 &= 0.519, \\
 RCC_4 &= 0.510.
 \end{aligned} \quad (46)$$

Step 6. Since $RCC_2 > RCC_3 > RCC_4 > RCC_1$, we can easily rank the preference order of the sellers as $S_2 > S_3 > S_4 > S_1$. Hence, the best seller is S_2 .

5.2. Comparison Analyses with Existing Methods Aggregated IF Information. Generally, the key focus of the heterogeneous MAGDM methods is how to aggregate the assessments taking the form of different data types, determine the weights of experts and attributes, and rank the preference order of alternatives. Here, we make detailed comparison analyses with some similar methods [34, 35, 37] from the abovementioned key issues, which are shown in Table 4. Furthermore, the current methodology has the following superiorities.

(1) A comparison between the current methodology and Yue's method [34] is made. We consider attribute weight vector $w = (0.4, 0.2, 0.4)$ and employ the former to evaluate the masses' satisfaction of three leaders used in [34]; the ranking of three suppliers is $S_1 > S_3 > S_2$ which is inconsistent with $S_1 > S_2 > S_3$ shown in [34]. Although the ranking is inconsistent, the best satisfactory leader is the same. Thus, the current methodology can well adapt to the special decision circumstance in [34]. However, the latter cannot deal with the heterogeneous MAGDM problems since it is only suitable for interval numbers. Moreover, in the former, the attribute weights are derived by using a multiple objective programming model that can avoid subjective randomness in the latter.

(2) A comparison between the current methodology and Xu et al. method [35] is made. Assume that $a = (\mu_1, v_1)$ and $b = (\mu_2, v_2)$ are two IFNs, $d(a, b) = (1/2)(|\mu_1 - \mu_2| + |v_1 - v_2|)$ is the distance of a and b , and $\alpha^+ = (1, 0)$ and $\alpha^- = (0, 1)$ are the largest and smallest IFN. Then, the former is applied to evaluate the cloud computing service provider used in [35]; the ranking of four suppliers is $S_1 > S_2 > S_3 > S_4$ which is inconsistent with $S_3 > S_1 > S_2 > S_4$ shown in [35]. Thus, the current methodology can well adapt to a special decision circumstance in [35]. The main reasons for the difference in the ranking are as follows. (i) The method for eliciting the satisfaction and dissatisfaction of DMs based on the TOPSIS idea in the current methodology is more complete than that in [35], because the former takes into account the two aspects of satisfaction and dissatisfaction in each element, while the latter considered only one aspect. (ii) Besides the rating values denoted by RNs, INs, TFNs, TrFNs, and LVs, the former can deal with more complicated rating values, including TIFNs, whereas the latter is only suitable for the HMAGDM problem with RNs, INs, TFNs, TrFNs, and LVs. Thus, the latter cannot solve the abovementioned example. (iii) In the former, the integrated information of experts is expressed by a TIFN, whereas that in the latter is an IFN. Hence, there are more opportunities resulting in a loss of information.

(3) A comparison between the current methodology and Wan et al. method [37] is made. We employ the former to evaluate the IT outsourcing service provider used in [37]; the ranking of four suppliers is $S_3 > S_2 > S_1 > S_4$ which is consistent with $S_3 > S_2 > S_1 > S_4$ shown in [37]. Thus, the current methodology can well adapt to a special decision circumstance in [37]. However, the current methodology is superior over the later in the following aspects. (i) In the

former, the integrated information of experts on the same attribute is a TIFN, whereas that in the latter is an IVIFN. Hence, the current methodology can express vagueness information of reality more accurately and abundantly. (ii) The former takes into account the weights of experts by their consistency degree, whereas there is no consideration in the latter. So, the current methodology is more reasonable. (iii) The same as the above, the latter is only suitable for the HMAGDM problem with RNs, INs, TFNs, and TrFNs. Thus, the latter cannot solve the abovementioned example.

5.3. Comparison Analyses with Existing HMAGDM Methods.

In this section, we compare the proposed method with other two methods for HMAGDM problems; one is the complex and dynamic MAGDM method developed by Dong et al. [15] that is an optimization-based approach, and the other is the GDM method based on integrating heterogeneous information introduced by Li [21] that is a direct approach. For simplicity, the comparative analysis methods are denoted as CD-GDM and IGI-GDM. The highlighted features of the proposed methods can be summarized as follows.

(1) During the initial phase, CD-GDM and IGI-GDM need to standardize the decision data, whereas there is no need for standardization in the proposed method which is relatively simple.

(2) The GDM matrix of the proposed method is an ITFN decision-making matrix containing ITFNs only which is easy to handle, whereas that of IGI-GDM remains a heterogeneous decision-making matrix which is difficult to deal with.

(3) In CD-GDM and IGI-GDM, the integrated information of experts is expressed by original decision data types and real number type which contains less information, while the integrated information of experts in the proposed method is represented by ITFNs which can express intuitively and describe satisfaction, dissatisfaction, and distribution of experts.

(4) The weights of attributes in IGI-GDM are given by decision makers in advance which are subjective; CD-GDM determine the weights of attributes by nonlinear programming model, whereas they construct a multiple objective linear programming model to establish attributes' weights. Thus the proposed method is more objective and effective.

6. Conclusions

In this paper, we put forward a new aggregation approach to solve such heterogeneous MAGDM problems in which the weights of the attributes are incompletely known. The key features of the proposed method are listed as follows: (1) a new similarity measure of ITFNs is proposed; (2) a new general approach to aggregating decision information into ITFNs is proposed. It can not only accommodate more complicated data of types, including INs, TFNs, TrFNs, and TIFNs, but also take importance of experts into account; (3) a new multiple objective mathematical programming model is developed for determining the attribute weights objectively under intuitionistic triangular fuzzy environment; (4) a new method is presented to solve heterogeneous MAGDM problems, which considers fully the indeterminacy of the

TABLE 4: Comparisons between the current method and existing methods.

Characteristics	Method [34]	Method [35]	Method [37]	The current method
Attribute values	Ins	RNs, Ins, TrFNs, TrFNs, IFNs, LVs	RNs, Ins, TrFNs, TrFNs,	RNs, Ins, TrFNs, TrFNs, LVs
Attribute normalizing	Need	Need	Need	No need
Solve problem	MAGDM	HMAGDM	HMAGDM	HMAGDM
Satisfactory information	Derived from $[0, 0.5]$	Based on elements satisfy $d(x_{ij}^k, A_j^{\max}) < d(x_{ij}^k, A_j^{\min})$	Based on all elements by TOPSIS	Based on all elements by TOPSIS
Dissatisfactory information	Derived from $[0.5, 1]$	Based on elements satisfy $d(x_{ij}^k, A_j^{\max}) > d(x_{ij}^k, A_j^{\min})$	Based on all elements by TOPSIS	Based on all elements by TOPSIS
Uncertain information	None	Based on distances from elements to the largest, middle and smallest grade	Based on distances from elements to the largest, middle and smallest grade	Based on geometry entropy method
Expert weight	None	None	None	Derived from consistency degree
Attribute weight	Given in advance	By multi-objective IF programming	By multi-objective IVIF programming	By multi-objective linear programming
Aggregated information	IFNs	IFNs	IVIFNs	ITFNs
Rank method	Based on score and accuracy function	Based on score and accuracy function	Based on score and accuracy function	Based on RCC

DMs in the assessment; thus the final decision results derived by the proposed method are more reasonable. Additionally, the proposed method can be also appropriate for the complex multiattribute large-group decision-making problems [51]. Future research will extend the developed method to heterogeneous MAGDM with complete unknown weight information under complex fuzzy environment. Meanwhile, as the scale of group increases and the decision makers have different backgrounds and levels of knowledge, it is difficult to achieve consensus among decision makers [52, 53]. Therefore, it will be very interesting in future studies to discuss the consensus reaching mechanism in the large-scale HMAGDM.

Data Availability

We state that the data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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